Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 1	DAILY TUTORIAL SHEET 2

26.(C) A, I, I, I, O can occur at odd places in $\frac{5!}{3!}$ ways, and the remaining letters N, D, L, L can be arranged at the remaining places in $\frac{4!}{2!}$ ways.

27.(D) We can arrange remaining 17 boys in 17! ways. For 3 particular boys we have 18 positions. We can arrange 3 particular boys at these places in ${}^{18}P_3$ ways. Thus, the required number of ways is $17! \times {}^{18}P_3 = \frac{17! \times 18!}{15!}$

28.(A) A, E, G, R, D, N
As vowels will be in alphabetical order A will come before E. First select 4 out of 6 places and arrange G, R, D and N in them in ${}^6C_4 \times \underline{|4}$ ways. For the remaining two places we have only 1 way of arranging A and E in them due to alphabetical order. $\Rightarrow$ The total number of ways = ${}^6C_4 \times \underline{|4} \times 1 = 360$

29.(A) B, A, A, A, N, N

Arrange one b and 3 A's in $\frac{\underline{|4}}{\underline{|3}} = 4$ ways.

Now arrange 2 N's in 5 places created by one B and 3 A's in ${}^5C_2$ ways

$\Rightarrow$ Total number of arrangements $= 4 \times {}^5C_2 = 40$

30.(A) Required no. of ways = ${}^{22}C_{19} = 1540$

31.(A) Arrange 5 negative signs around a circle in one way and there will be 5 gaps in between them. Four "+" signs can be filled in those five gaps in exactly one way.

32.(A) We can choose 10 players from $(22-2)$ players in ${}^{20}C_{10}$ ways and one player from 2 players in ${}^2C_1$ ways. $\therefore$ Required number of ways $= \frac{20!}{10!\,10!} \times 2 = 369512$

33.(C) There are four even places for the four odd digits 3, 3, 5, 5

$\therefore$ The required no. of ways $= \frac{4!}{2!2!} \cdot \frac{5!}{2!\,3!} = 60$.

34.(A) *RR, A, A, N, G, E*

$|A\ |A\ |N\ |G\ |E\ |$ Number of words = $\frac{5!}{2!} \times {}^6C_2 \times \frac{2!}{2!}$

$\frac{5!}{2!}$	${}^6C_2$	$\frac{2!}{2!}$
No. of ways to arrange A, A, N, G, E.	No. of ways to select 2 places for 2 R's	No. of ways to arrange 2 R'S

35.(B) *I, N, T, G, R, EE*

m_1: $|\,T\,|\,G\,|\,R\,|\,E\,|\,E$ $\quad m_1 = \frac{5!}{2!} \times {}^6C_2 \times 2!$

m_2: I _ _ _ _ _ _ _R $\quad m_2 = \frac{5!}{2!}$ $\quad \Rightarrow \quad m_1/m_2 = {}^6C_2 \times 2! = 30$

36.(D) The required number of ways = the number of ways in which 8 girls can sit in a row – the number of ways in which two sisters sit together = $8! - (2)(7!) = 30240$.

37.(B) The word MATHEMATICS contains 11 letters viz. M, M, A, A T, T, H, E, I, C, S. The number of words that begin with T and end with T is $\frac{9!}{2!2!} = 90720$.

38.(C) k diff. things taken not more than 'r' at a time implies $\underset{\underset{k}{\Downarrow}}{1 \text{ objects}} + \ldots\ldots \underset{\underset{k^r}{\Downarrow}}{r \text{ objects}} \equiv k\dfrac{(k^r - 1)}{k-1}$

39.(D) – – – – 2 vowels, 2 consonants

(1) Select two places out of 4 in $^4C_2$ ways for 2 consonants and arrange them in 21 × 21 ways [∵ 21 consonants are there, and repetition is allowed]

(2) Arrange 2 vowels out of 5 on remaining two places in $^2C_2 \times 5 \times 5$ ways

Total number of ways = $^4C_2 \times 21^2 \times {}^2C_2 \times 5^2 = 6 \times 105^2$

40.(A) The required no. of ways = number of non-negative solutions of $x_1 + x_2 + x_3 + x_4 = 6 = {}^9C_3$.

41.(A) For one letter we have three options of boxes.

∴ Total number of ways = $3 \times 3 \times 3 \times 3 \times 3$.

42.(C) Out of 9 men two men can be chosen in $^9C_2$ ways. Since no husband and wife are to play in the same game, so, we have to select two women from the remaining 7 women. This can be done in $^7C_2$ ways. If M_1, M_2, W_1, W_2 are chosen, then a team can be constituted in 2 ways. Thus, the number of ways of arranging the game = $^9C_2 \times {}^7C_2 \times 2 = 1512$.

43.(B) Total number of books $= a + 2b + 3c + d$

Since there are b copies of each of two books, c copies of each of three books and single copies of d books. Therefore, the total number of arrangements is $\dfrac{(a+2b+3c+d)!}{a!\,(b!)^2(c!)^3}$.

44.(B) Required ways = (Exactly 2) + (Exactly 3) + ... + (Exactly n – 2)

= Total ways – (Exactly 0) – (Exactly one) – (Exactly n – 1) – (Exactly n)

$= 2^n - {}^nC_0 - {}^nC_1 - {}^nC_{n-1} - {}^nC_n = 2^n - 2 - 2n$

45.(A) $x_1 + x_2 + x_3 + x_4 + x_5 = 15$; where x_i means number of balls in Box-i

$x_i \geq 2$ as per question

Let $x_i = y_i + 2 \quad \Rightarrow \quad y_1 + y_2 + y_3 + y_4 + y_5 = 5;\ y_i \geq 0 \quad \Rightarrow \quad {}^{5+5-1}C_{5-1} = {}^9C_4 = {}^9C_5$

46.(C) Answer = $^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 = 70 + 56 + 28 + 8 + 1 = 163$

47.(B) The number of choices available to him.

$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5 = \dfrac{5!}{4!\,1!} \times \dfrac{8!}{6!\,2!} + \dfrac{5!}{5!\,0!} \times \dfrac{8!}{5!\,3!} = 5 \times \dfrac{8 \times 7}{2} + 1 \times \dfrac{8 \times 7 \times 6}{3 \times 2}$

$= 5 \times 4 \times 7 + 8 \times 7 = 140 + 56 = 196$

48.(A) *T, E, X, M, O, A, A, II, NN*

(a) All alike (Not possible)

(b) 3 alike, 1 diff (Not possible)

(c) 2 alike, 2 alike $\equiv {}^3C_2 \times 4! / 2!\,2! = 18$

(d) 2 alike, 2 diff $^3C_1\ {}^7C_2 \times 4! / 2! = 756$

(e) All diff. $^8C_4 \times 4! = 1680$

Total ways = 18 + 756 + 1680 = 2454

49.(B) *PP, RR, OOO, T, N, I*

(a) All alike (Not possible)

(b) 3 alike, 1diff. $^1C_1\ {}^5C_1 \times 4! / 3! = 20$

(c) 2 alike, 2 alike $\equiv {}^3C_2 \times 4! / 2!\,2! = 18$

(d) 2 alike, 2 diff. $\equiv {}^3C_1 \times {}^5C_2 \times 4! / 2! = 360$

(e) All diff. $\equiv {}^6C_4 \times 4! = 360$

Total ways = 20 + 18 + 360 + 360 = 758

50.(B) *AAA, SSSS, II, NN, T, O* ← Question - 6 is selection

(a) All alike $\equiv {}^1C_1 \times 4! / 4! = 1$

(b) 3 alike, 1 alike $\equiv {}^2C_1 \times {}^5C_1 \times 4! / 3! = 40$

(c) 2 alike, 2 alike $\equiv {}^4C_2 \times 4! / 2!\,2! = 36$

(d) 2 alike, 2 diff. $\equiv {}^4C_1\ {}^5C_2 \times 4! / 2! = 480$

(e) All diff. $\equiv {}^6C_4 \times 4! = 360$

Total number of words = 1 + 40 + 36 + 480 + 360 = 917